F1

The graph of the function $f(x) = x^3 - 4x$ is given in Figure 9 (on p14).

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F2

$H(x) = x^2 - 4x + 6 = x^2 - 4x + 4 + 2 = (x - 2)^2 + 2.$

This last expression is the completed square form. The vertex is situated at the point (2, 2). The graph does not intersect the x-axis, because the discriminant $b^2 - 4ac$ is negative. $H(x)$ does not have an inverse function because its graph is a parabola, so different values of x do not necessarily correspond to different values of $H(x)$.

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F3

As x approaches 1 from above (i.e. decreasing from even greater values of x), $g(x)$ becomes a larger and larger positive number. As x approaches 1 from below (i.e. from lower values of x), $g(x)$ becomes a larger and larger negative number. Thus $x = 1$ is an asymptote. When x is a large number, positive or negative, $g(x) \approx x^2/x = x$. Thus the line $y = x$ is also an asymptote. These are the only asymptotes, as you can confirm by sketching the graph.

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R1

All the numbers are *real numbers*; 2 and 7 are *integers*

(7.0 is ambiguous, but it would not be generally regarded as an integer).

(If you are unclear about these terms see the *Glossary*.)

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R2

(a), (b), (c) an (d) are all true but (e) is false.

Note that the *modulus* (or *absolute value*) of any quantity, $|x|$, is never negative. For the purposes of these comparisons any negative number is regarded as being less than any positive number, and negative large numbers are regarded as being less than negative small numbers.

(See *number line* in the *Glossary*.)

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R3

(a) $5^2 = 25$

(b) $(-4)^3 = -64$

(c) $\sqrt{49} = \pm 7$ (note that in *FLAP* *square roots* are positive *or* negative)

(d) $\sqrt[3]{-2\,000} = -12.60$ (to two *decimal places*)

(e) $\sqrt{0.09} = \pm 0.3$ (to one *significant figure*)

(f) $|5| = 5$

(g) $|-3.2| = 3.2$.

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R4

(a) $a^2 \times a = a^{2+1} = a^3$, (b) $a^3 \times a^2 = a^{3+2} = a^5$, (c) $b^3/b^2 = b^{3-2} = b$, (d) $c^4 \times c^2/c^6 = c^{4+2-6} = c^0 = 1$.

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R5

$a^2 + 3a - 4a + 7 = a^2 - a + 7 = a(a - 1) + 7.$

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R6

(a) $(u + 3)(u - 3) = u^2 + 3u - 3u - 9 = u^2 - 9$.

(b) $(p + 2)(p - 4) + (p + 1)^2 = p^2 - 4p + 2p - 8 + p^2 + 2p + 1 = 2p^2 - 7$.

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T1

Yes. There is a single definite value of your height for every value of the date. The fact that it may be the same value for many different dates doesn't matter.

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T2

Yes. There are two possible values, 0 and 1. For some dates (in the winter) the value will be 0 (i.e. the sun doesn't rise), for others (during the rest of the year) the value will be 1.

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T3

No. Simply being told the time of day does not determine the date. (Being told the time of some particular daily event might allow you to determine the date, but the time alone is not enough, the same times recur every day.)

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T4

D is (–2, 1), E is (–2, –2), F is (–1, –1), G is (1, –2) and O is (0, 0).

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T5

See Figure 15.

There are real discontinuities where the rate changes, so you should not try to join the lines up.

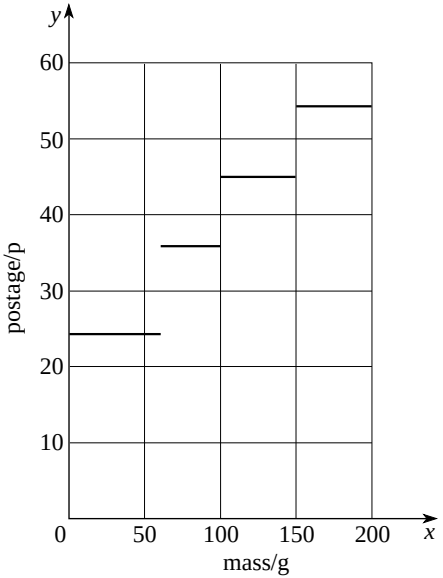

Figure 15 See Answer T5.

T6

See Figure 16.

Draw the curve ‘with a flowing hand’.

(It does not pass through the origin.)

The points supplied by Table 1 have been plotted as a guide to the curve.

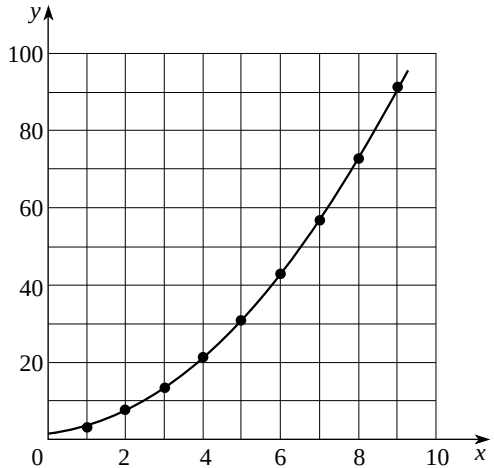

Figure 16 See Answer T6.

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T7

$y - y_0 = m(x - x_0)$, therefore $y - y_0 = mx - mx_0$

So, $y = mx - mx_0 + y_0$, hence the gradient is m and the intercept $y_0 - mx_0$.

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T8

First of all note that, from Equation 6, the right-hand side of Equation 8 is just equal to m. So we can write Equation 8 as $(y - y_1)/(x - x_1) = m$.

If this is then rearranged, $y - y_1 = m(x - x_1)$, which is in the same form as Equation 7.

Therefore the answer follows from Answer T7, the intercept is $y_1 - mx_1$.

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T9

When $x = 0$, $y/b = 1$, so $y = b$.
When $y = 0$, $x/a = 1$, so $x = a$.
Hence a and b are the intercepts where the straight line cuts the x- and y-axes, respectively.

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T10

The graphs of $k(x)$ and $g(x)$ are sketched in Figure 17.

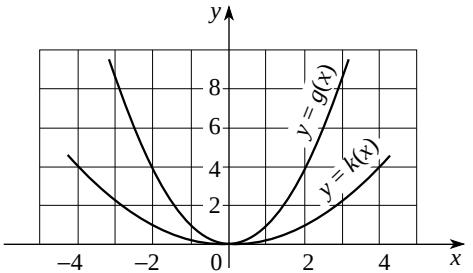

Figure 17 See Answer T10.

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T11

(a) As you know the vertex of $a(x - p)^2$ is at $(p, 0)$ you should be able to see that the required quadratic function has the form

$$f(x) = a(x - p)^2 + q$$

since the additional q will have the effect of moving each point on the parabola, including the vertex, q units upwards.

(b) The quadratic functions (and vertices) are:

(i) $f(x) = (x - 1)^2 + 3$ vertex at (1, 3)

(ii) $f(x) = (x + 2)^2 + 1$ vertex at (–2, 1)

iii) $f(x) = (x - 2)^2 - 1$ vertex at (2, –1)

They are sketched in Figure 18. Note that each has the same 'width' since a is the same in each case.

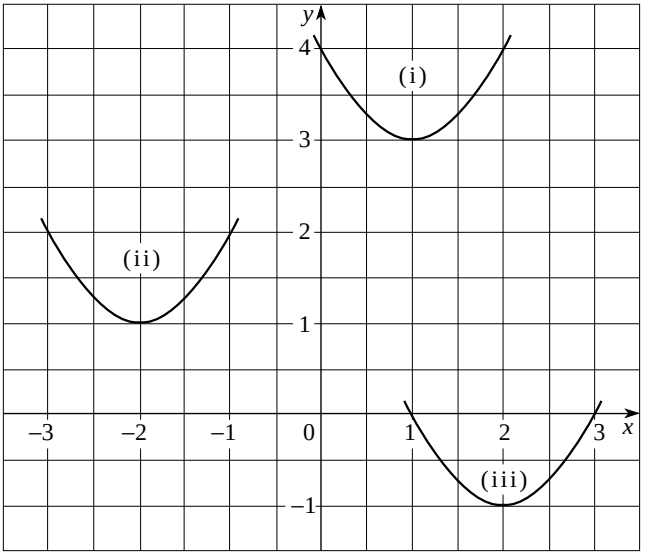

Figure 18 See Answer T11.

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T12

(a) $f(x) = 3x^2 - 9x + 11$ does not intersect the x-axis since its discriminant is negative. In completed square form (see Equation 14)

$$f(x) = 3\left(x - \frac{3}{2}\right)^2 + \frac{17}{4}$$

So its vertex (a minimum) is at (3/2, 17/4).

(b) $f(t) = -t^2 - 2t - 6$ does not intersect the t-axis since its discriminant is negative. In completed square form

$$f(t) = -(t + 1)^2 - 5$$

so its vertex (a maximum) is at (–1, –5).

(c) $f(R) = -3R^2 + 15R - 18$ intersects the R-axis twice since its discriminant is greater than zero. In completed square form

$$f(R) = -3\left(R - \frac{5}{2}\right)^2 + \frac{3}{4}$$

so its vertex (a maximum) is at (5/2, 3/4).

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T13

(a) $y = \dfrac{-5 \pm \sqrt{25 + 4}}{2} = -5.193$ and 0.193.

(b) The *discriminant* is less than zero, so there are no points of intersection.

(c) $Z = \dfrac{-0.5 \pm \sqrt{0.25 + 3}}{6} = 0.217$ and -0.384.

(d) $x = \dfrac{5 \pm \sqrt{25 - 24}}{2} = 3$ and 2.

In simple cases such as (d) it is not always necessary to use the formula to find the points of intersection.

With practice you should be able to see 'by inspection' the factorized form of the function:

$x^2 - 5x + 6 = (x - 2)(x - 3)$.

Practice at developing this skill is given elsewhere in *FLAP*.

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T14

(a) See Figure 19. The asymptotes are the lines $y = 1$ and $x = -1$, i.e. as $x \to \infty$, $y \to 1$ and when $x = -1$ the denominator = 0.

(b) $g(x)$ also has a vertical asymptote, at $x = -1/3$, but there is no horizontal asymptote. Instead, as $x \to \pm\infty$, $g(x) \approx 2x^2/(3x) = 2x/3$. So, the line $y = 2x/3$ is also an asymptote.

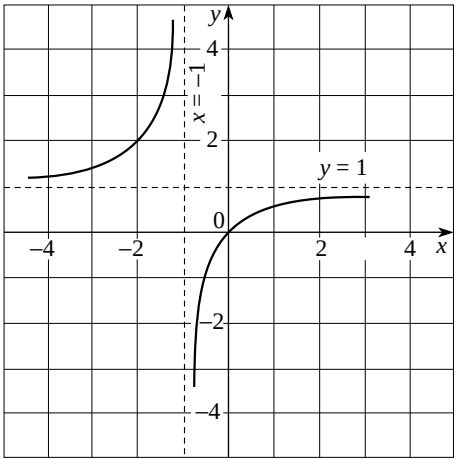

Figure 19 See Answer T14.

T15

The graph of Figure 8 shows that each value of y corresponds to a unique value of x. Thus there is no difficulty about defining the inverse function of Equation 20.

$$f(x) = x^3 \qquad \text{(Eqn 20)}$$

In fact it's just $G(x) = \sqrt[3]{x}$.

Figure 9 shows a different situation. Each value of y does not correspond to a unique value of x, so it is not possible to define an inverse function.

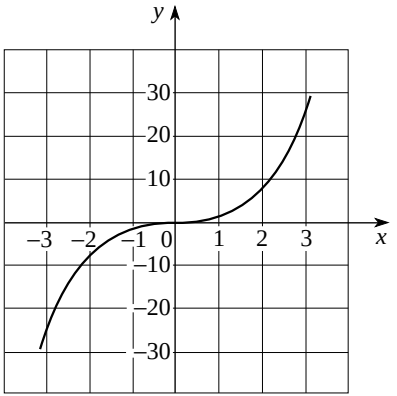

Figure 8 The cubic function $f(x) = x^3$.

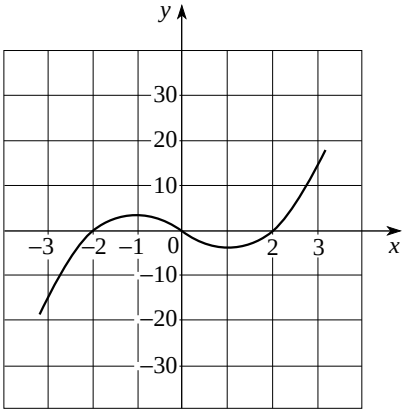

Figure 9 The cubic function $g(x) = x^3 - 4x$.

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T16

No. A particular value of $f(x)$ may correspond to more than one value of x, so it is not possible to find an inverse function.

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T17

$g(f(x)) = g(x + 2) = 1/(x + 2)$ where x is any real number except -2.

$f(g(x)) = f(1/x) = (1/x) + 2$ where $x \neq 0$.

Be careful to carry out the operations step by step and in the right order.

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T18

$g(h(f(y))) = g(h(y + 2)) = g((y + 2)^2) = 1/(y + 2)^2$ where $y \neq -2$.

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T19

$G(F(x)) = x$ since $G(x)$ 'undoes' the effect of $F(x)$ for every value of x in the domain of $F(x)$. More formally, if we let $y = F(x)$ and $x = G(y)$,

$$H(x) = G(F(x)) = G(y) = x.$$

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