If you have already studied one of the *FLAP* modules, you may have expected to find some ‘Ready to study’ questions here. However, as this is an introductory module there are none in this case.

$Note$: $3a = 3 \times a$

The symbol ≠ means ‘is *not* equal to’.

The convention used in *FLAP* for the order of brackets is as follows:

$$\{[(x)]\}$$

( ) are called parentheses

[ ] are called square brackets

{ } are called braces

Powers, roots and reciprocals are the subject of Subsection 2.4.

When expanding brackets take particular care over negative terms. Even if you remember to multiply 4 by (–3) to get the third term you may still forget to multiply (–3) by (–5) to get the fourth term.

You may think that using brackets to group terms together makes things look more complicated, rather than simpler, but, as you will see in the rest of this module, it can be a useful step towards simplifying fractions and equations.

To three decimal places
$\pi = 3.142$.

Though $(1/x) + a$ would have been right

Strictly speaking, the second, third and fourth of these expressions are ambiguous and need brackets to make them unambiguous. However, many would take the view that the style in which they are printed makes clear the presumed order of the various divisions

It is easy to misremember the algebraic rules for dealing with fractions. If in doubt, try out the method you *believe* to be correct with some simple numbers and check that it works before applying it to anything more complicated.

The relation $a^0 = 1$ may look strange, but it is a logical step in the pattern.

$8^{1/3}$ is the cube root of 8.

The peculiar properties of roots make more sense when discussed in the context of *complex numbers* — the subject of other *FLAP* modules. In this module all references to ‘numbers’ relate to ordinary (real) numbers and specifically exclude complex numbers.

Roots of negative quantities may be defined in the context of *complex numbers*, but we are ignoring that possibility in this module.

Although these equations are similar to Equations 9 and 10, p and q have been used to represent the powers so that you will no longer think of them as positive whole numbers. They may be negative, fractional or even numbers such as π that can be represented as infinite decimals but not as finite fractions.

ρ, σ and η are the Greek letters ‘rho’, ‘sigma’ and ‘eta’, respectively.

Many physicists would be reluctant to take this last step since it probably makes the physical interpretation of F less obvious.

Although this is a bogus proof, note the way it is laid out. Clear layout is always helpful, especially if you have to find an error, as in this case.

Note that the symbol $\propto$ is *not* the same as the Greek letter alpha α, take care to make them look different when you write them.

$k = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$ is known as *Boltzmann's constant*.

In reading this subsection you may be wondering what distinguishes a ‘true’ constant from a variable that is held constant? This is a deep question to which there is no simple answer. It has been suggested (notably by Paul Dirac (1902–1984)) that some of the fundamental constants of physics might actually change very gradually with time.

✧

(a) Yes, $ab = ba$
for every choice of a and b

(b) Yes, $a + b = b + a$
for every choice of a and b

(c) No, $a - b \neq b - a$
for every choice of a and b

(d) No, $a/b \neq b/a$
for every choice of a and b. ❑

✧

(a) $-12x + 6$ (each term in the bracket is multiplied by -3)

(b) $2(3x + 2y) - 4y = 6x + 4y - 4y = 6x$

(c) $3(x + 2(2x + 5)) = 3(x + 4x + 10)$
$= 3(5x + 10) = 15x + 30.$ ❑

✧

(a) $$\frac{3}{5} \div \frac{5}{2} = \frac{3 \times 2}{5 \times 5} = \frac{6}{25} = 0.24$$

(b) $$\frac{mv^2}{2} \div \frac{r}{2} = \frac{mv^2 \times 2}{2 \times r} = \frac{mv^2}{r}$$ ❑

✧

$$\frac{3x}{5} = \frac{3xy}{5y} \quad \text{and} \quad \frac{2}{y} = \frac{10}{5y}$$

Thus

$$\frac{3x}{5} - \frac{2}{y} = \frac{3xy}{5y} - \frac{10}{5y} = \frac{3xy - 10}{5y}.$$

❑

✧

You have to *reduce* the power by 1, i.e. *divide* the previous number by *a*, or 5 in this case. ❑

✧

The reciprocal of a^2 may be written as $1/a^2$ or $(a^2)^{-1}$. ❑

✧

a^p divided by a^q is identical to a^p multiplied by a^{-q}.

So, $\quad a^p / a^q = a^p \times a^{-q} = a^{p-q}$

where we have used the rule for multiplication with q replaced by $-q$. ❑

✧

If we use the general rules we may equally well write

$$a^{p/q} = (a^p)^{1/q} \text{ or } a^{p/q} = (a^{1/q})^p$$

so $(a^p)^{1/q}$ is the qth root of the pth power a

and $(a^{1/q})^p$ is the pth power of the qth root of a. ❑

✧

$$F = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g - 6\pi\eta r v$$

If a common factor of $\frac{4}{3}\pi r^3 g$ is extracted from the first two terms on the right

$$F = \frac{4}{3}\pi r^3 g(\rho - \sigma) - 6\pi\eta r v$$

As a last step it is possible to extract a common factor of $2\pi r$ from each term on the right-hand side ☞

$$F = 2\pi r\left[\frac{2}{3}r^2 g(\rho - \sigma) - 3\eta v\right] \qquad (16)$$

❑

✧

From a mathematical point of view this is a ‘silly’ proof from the outset, but there is not actually anything wrong with it until both sides are divided by $(a - b)$. Since $a = b$, this step amounts to dividing both sides by zero, which is not a legitimate procedure. The result of the division is quite bogus, as you can see by replacing a and b by some simple number such as 1 throughout the ‘proof’. ❑

✧

Using the values chosen earlier

$$\frac{v^2}{2s} = \frac{28}{8} = 3.5, \text{ but } u^2 + a = 4 + 3 = 7$$

Since the same set of values cannot satisfy the original equation and its supposed rearrangement there is clearly something amiss. ❑

✧

> means ‘is greater than’ and ≥ means ‘is greater than or equal to’. ❑

✧

$3 \leq x \leq 10$

(Note that in writing statements of this kind it is conventional to put the lesser value on the left — just like the number line.) ❑

✧

(a) Dividing by $(1 - a)$ when $a = 1$ is just another way of dividing by zero. It will produce meaningless results and should be avoided.

(b) Multiplying both sides of an inequality by zero is also dangerous. It's true that $3 < 6$, but is *not* true that $(0) \times (3) < (0) \times (6)$, i.e. $0 < 0$.

(c) Even if $a > b$ it is *not* generally true that $a^2 > b^2$. For instance $-3 > -4$ but it is *not* true that $(-3)^2 > (-4)^2$.

(d) Square roots also need care. Negative quantities don't have square roots and positive quantities have two square roots, so although $9 > 4$, it is *not* true that $-3 > -2$ though it *is* true that $3 > 2$. ❑