F1

The proof of this very important result is given in Subsections 3.1 and 3.3 and is illustrated in Figures 6, 7 and 8. Compare this discussion with your own answer and revise the relevant subsections if necessary. In brief, the argument is first to show that a body in uniform circular motion has an acceleration towards the centre of the orbit. This is true both for the average acceleration between any two different points on the path and also for the instantaneous acceleration at any point on the path. This proof involves taking the limit as the interval tends to zero, and using the calculus notation. As a second stage, the magnitude of the acceleration (centripetal acceleration) is shown to equal v^2/r or the two equivalent expressions $v\omega$ or $r\omega^2$. This is an important argument and you should be sure that you are happy with it.

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F2

We are given the speed v and the magnitude of the centripetal force F for the planet. We can calculate its angular speed ω from the orbital period T using

$$\omega = 2\pi / T \qquad \text{(Eqn 5)}$$

In this case

$$\omega = \frac{2\pi}{2.6 \times 10^{7}} \text{ rad s}^{-1} = 2.4 \times 10^{-7} \text{ rad s}^{-1}$$

We can now calculate m using a rearrangement of Equation 10a

$$F = ma = mv\omega \qquad \text{(Eqn 10a)}$$

$$m = F/v\omega$$

Hence $$m = \frac{3.0 \times 10^{22}}{2.0 \times 10^{5} \times 2.4 \times 10^{-7}} \text{ kg} = 6.3 \times 10^{23} \text{ kg}$$

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The radius of the orbit can be found from Equation 10c

$$F = ma = mr\omega^2 \qquad \text{(Eqn 10c)}$$

$$r = F/m\omega^2$$

So, $$r = \frac{3.0 \times 10^{22}}{6.3 \times 10^{23} \times \left(2.4 \times 10^{-7}\right)^2}\ \text{m} = 8.3 \times 10^{11}\ \text{m}$$

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R1

(a) If the *acceleration* is uniform, its *magnitude* a can be expressed as

$$a = \frac{\Delta v}{\Delta t}$$

where Δv is the magnitude of the change in *velocity* during the time interval Δt

$$a = \frac{10.0 - 5.00}{10.0}\,\mathrm{m\,s^{-2}} = 0.500\,\mathrm{m\,s^{-2}}$$

(b) For uniform acceleration, the *average speed* $\langle v \rangle$ in the time interval Δt is given by

$$\langle v \rangle = \frac{v_0 + v_1}{2}$$

where v_0 and v_1 are the initial and final speeds, respectively. So for this case,

$$\langle v \rangle = \frac{5.00 + 10.0}{2}\,\mathrm{m\,s^{-1}} = 7.50\,\mathrm{m\,s^{-1}}$$

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(c) The *distance* travelled is

$$\Delta s = \langle v \rangle \Delta t = (7.50 \times 10.0)\,\mathrm{m} = 75.0\ \mathrm{m}$$

(If you are unclear about any of these terms consult the *Glossary*.)

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R2

(a) The *average speed* is $\langle v \rangle = \Delta s/\Delta t$

where Δs is the distance travelled in the time interval Δt.

In this case $\langle v \rangle = \dfrac{270 - 40}{10}\,\mathrm{m\,s^{-1}} = 23\,\mathrm{m\,s^{-1}}$

(b) The *instantaneous speeds* at $t = 5\,\mathrm{s}$ and $t = 15\,\mathrm{s}$ are obtained by drawing *tangent lines* to the graph at these points. This gives the following results

for $t = 5\,\mathrm{s}$ $\quad v_5 = 12\,\mathrm{m\,s^{-1}}$

for $t = 15\,\mathrm{s}$ $\quad v_{15} = 36\,\mathrm{m\,s^{-1}}$

(c) The *average acceleration* in this time interval is

$$\langle a \rangle = \frac{|\Delta \boldsymbol{v}|}{\Delta t} = \frac{|\boldsymbol{v}_{15} - \boldsymbol{v}_5|}{\Delta t} = \frac{36 - 12}{10}\,\mathrm{m\,s^{-2}} = 2.4\,\mathrm{m\,s^{-2}}$$

Comment Because these values are calculated from a graph, your answers to parts (b) and (c) may differ from those given here. Values within 10% of the quoted figures are acceptable. (If you are unclear about any of these terms consult the *Glossary*.)

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R3

The value of an angle in *radians* can be calculated from the relationship $s = r\theta$ or $\theta = s/r$, where s is the *arc length* and r is the radius.

In this case $\theta = \left(\frac{10}{12}\right)\text{rad} = 0.83\,\text{rad}$

Comment

If you are unclear about any of these terms consult the *Glossary*.

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R4

The addition of $\boldsymbol{a}$ and $\boldsymbol{b}$ is carried out according to the usual *triangle rule for the addition of vectors*. This is shown in Figure 9.
From Figure 9, the *vector* $\boldsymbol{c}$ has a *magnitude* of 5 units and a direction 37° East of North. This can be determined either by measurement from the figure or from *Pythagoras's theorem* and *trigonometry*. Thus

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2} = \sqrt{4^2 + 3^2} = 5$$

and $$\sin\theta = \frac{b}{c} = \frac{3}{5}$$

Therefore $\theta = 37°$

Comment More information on this topic is contained in the maths strand of *FLAP*. (If you are unclear about any of these terms consult the *Glossary*.)

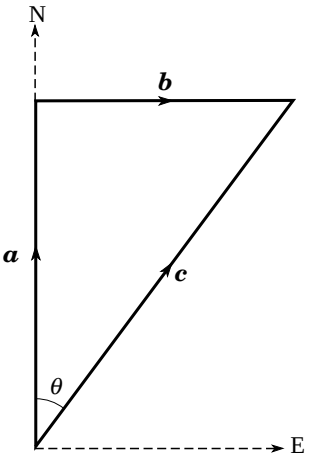

Figure 9 See Answer R4.

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T1

From Equation 4a,

$$\langle \omega \rangle = \frac{\theta}{t} \qquad \text{(Eqn 4a)}$$

the unit of angular speed should be the unit of angle divided by the unit of time. In the SI system, the standard unit of angle is the radian and the standard unit of time is the second. The unit of angular speed is therefore radians per second (rad s^{-1}).

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T2

(a) For the spin drier there are 3500 revolutions of the spin drier drum in one minute which means there are 3500/60 revolutions in one second. But there are 2π radians per revolution.

So, $\omega = \dfrac{3500 \times 2\pi}{60}\ \mathrm{rad\,s^{-1}} = 366.5\ \mathrm{rad\,s^{-1}}$

(b) The speed of the sock is given by $v = r\omega$

So, $v = 0.25\ \mathrm{m} \times 366.5\ \mathrm{rad\,s^{-1}} = 92\ \mathrm{m\,s^{-1}}$

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T3

(a) For a point on the surface of the Earth, the angular speed can be calculated from the period, which is of course 24 hours.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600}\ \text{rad s}^{-1} = 7.27 \times 10^{-5}\ \text{rad s}^{-1}$$

The corresponding speed at the Equator is given by $v = r\omega$

$$v = 6.38 \times 10^{6}\ \text{m} \times 7.27 \times 10^{-5}\ \text{rad s}^{-1} = 4.64 \times 10^{2}\ \text{m s}^{-1}$$

(b) For the centre of the Earth in its orbit, we repeat the calculations, this time with $T = 1$ year and $r = 1.50 \times 10^{11}$ m.

$$\omega = \frac{2\pi}{365 \times 24 \times 3600}\ \text{rad s}^{-1} = 1.99 \times 10^{-7}\ \text{rad s}^{-1}$$

and $\quad v = r\omega, \quad$ so

$$v = 1.50 \times 10^{11}\ \text{m} \times 1.99 \times 10^{-7}\ \text{rad s}^{-1} = 2.99 \times 10^{4}\ \text{m s}^{-1}$$

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T4

This is not a contradiction. You should recall that in physics ‘acceleration’ is defined as the rate of change of velocity with time. Now velocity is a vector, so its direction is important as well as its magnitude. Thus an increase in speed, a decrease in speed or just a change in direction at constant speed can all be described as ‘accelerations’.

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T5

The relationship between $|\Delta \boldsymbol{v}_{AB}|$, v and $\Delta\theta$ is the same as the relationship between s, r and θ in Equation 2.

$$s = r\theta \quad (\theta \text{ in radians}) \qquad \text{(Eqn 2)}$$

That is

$$|\Delta \boldsymbol{v}_{AB}| = v\Delta\theta$$

This can also be seen from the geometry of the (*isoceles*) triangle in Figure 8:

$$|\Delta \boldsymbol{v}_{AB}| = \Delta v_{AB} = 2v \sin(\Delta\theta/2) \approx 2v(\Delta\theta/2) = v\Delta\theta$$

(since $\Delta\theta$ is small)

(This will become Equation 7.)

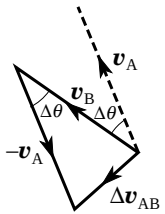

Figure 8 Triangle rule for the subtraction of v_A from v_B.

It might worry you that s in Equation 2 is an *arc* length whereas Δv_{AB} is the magnitude of a vector which is represented as a *straight line distance* in the diagram, but the small size of the angle $\Delta\theta$ allows us to do this. As $\Delta\theta$ gets smaller and smaller, the arc length and the straight line distance become more and more nearly equal and in the limit as $\Delta\theta$ approaches zero, we can regard them as being the same.

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T6

If we substitute $\omega = v/r$ in Equation 9a

$$a = v\frac{d\theta}{dt} = v\omega \qquad \text{(Eqn 9a)}$$

$$a = v\omega = v\left(\frac{v}{r}\right) = \frac{v^2}{r}$$

(This will become Equation 9b.)

If we substitute $v = r\omega$ in Equation 9a

$$a = v\omega = (r\omega)\omega = r\omega^2$$

(This will become Equation 9c.)

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T7

From Answer T3 we already have a value of $\omega = 7.27 \times 10^{-5}\ \mathrm{rad\,s^{-1}}$ for a point on the Earth's surface.

From Equation 9c

$$a = r\omega^2 \qquad \text{(Eqn 9c)}$$

$$a = 6.38 \times 10^6 \times (7.27 \times 10^{-5})^2\ \mathrm{m\,s^{-2}}$$

$$a = 3.37 \times 10^{-2}\ \mathrm{m\,s^{-2}}$$

From Newton's second law the magnitude of the force is given by

$$F = ma$$

Therefore, an 80 kg man standing on the Earth's Equator will experience a centripetal force of magnitude

$$F = 80 \times 3.37 \times 10^{-2}\ \mathrm{N} = 2.70\ \mathrm{N}$$

Both force and acceleration are directed towards the centre of the Earth. (Specifying the direction is a *vital* part of the answer.) The centripetal force is provided by the gravitational attraction of the Earth.

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T8

(a) From Answer T2, $r = 0.25\,\text{m}$ and $\omega = 367\,\text{rad s}^{-1}$ for the spin drier drum. If the mass of a sock is estimated to be 0.05 kg, we have

$$a = r\omega^2 \qquad \text{(Eqn 9c)}$$

$$a = 0.25 \times (367)^2\,\text{m s}^{-2} = 3.4 \times 10^4\,\text{m s}^{-2}$$

and $F = ma = 0.05 \times 3.4 \times 10^4\,\text{N} = 1.7 \times 10^3\,\text{N}$ (two significant figures).

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(b) Suppose the rubber had a mass of 0.1 kg, its orbital period about 0.5 s, and radius of the orbit about 0.3 m.

If we use $a = r\omega^2$ (Eqn 9c)

with $\omega = 2\pi/T$ (Eqn 5)

we get $a = \dfrac{4\pi^2 r}{T^2} = \dfrac{4\pi^2 \times 0.3}{(0.5)^2}\,\mathrm{m\,s^{-2}} = 47\,\mathrm{m\,s^{-2}}$

and the force (only justified to one significant figure)

$$F = ma = 0.1 \times 47\,\mathrm{kg\,m\,s^{-2}} = 4.7\,\mathrm{N} \approx 5\mathrm{N}$$

Comment

Your experimental values will probably differ from those given in (b).

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T9

Both students are partly correct although neither is telling the whole story. What the first student should say is, 'If $a = v^2/r$ then a will decrease with increasing r *provided* v *remains constant*.' The second student should say 'If $a = r\omega^2$, a will increase with increasing r *provided* ω *remains constant*.' The statements are not contradictory as it is impossible for *both* v and ω to remain constant if r is increasing. This follows from the relationship $v = r\omega$. If v is to remain constant as r increases, then ω must decrease proportionately. Similarly if ω is to remain constant, v has to increase.

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T10

If we rearrange Equation 10b

$$F = ma = \frac{mv^2}{r} \qquad \text{(Eqn 10b)}$$

it gives the speed as

$$v = \sqrt{\frac{rF}{m}}$$

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T11

From Answer T10

$$v = \sqrt{\frac{rF}{m}} = \sqrt{ra}$$

where a is the acceleration experienced by the body.

For the snowball, the acceleration is the acceleration due to gravity close to the Earth's surface. The required orbital speed is

$$v = \sqrt{6.38 \times 10^6 \times 9.8}\ \mathrm{m\,s^{-1}} = 7.9 \times 10^3\ \mathrm{m\,s^{-1}}$$

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T12

Equation 13 gives the relationship between speed and radius, just at the point of skidding

$$v_{\max} = \sqrt{r\mu g} \qquad \text{(Eqn 13)}$$

$$v_{\max} = \sqrt{3.0 \times 10^2 \times 0.40 \times 9.8}\ \mathrm{m\,s^{-1}} = 34\ \mathrm{m\,s^{-1}}$$

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T13

From Equation 16

$$v = \sqrt{\frac{Gm_2}{r}} \qquad \text{(Eqn 16)}$$

$$v = \sqrt{\frac{6.67\times10^{-11}\times5.98\times10^{24}}{1\times10^{6}+6.38\times10^{6}}}\ \mathrm{m\,s^{-1}}$$

$$= 7.35\times10^{3}\ \mathrm{m\,s^{-1}}$$

Notice that this speed is slightly *less* than the orbital speed needed for the snowball in Answer T11. This is because the acceleration due to gravity at the satellite's orbit is *less* than that at the Earth's surface, or equivalently that the orbital radius is *larger* for the satellite than for the snowball.

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T14

Rearrange Equation 17 as before

$$\omega = \frac{v}{r} = \sqrt{\frac{Gm_2}{r^3}} \qquad \text{(Eqn 17)}$$

$$r = \sqrt[3]{\frac{Gm_2}{\omega^2}}$$

and then, using the result for the angular speed of the Earth from Answer T3, substitute the appropriate values for Mars

$$r = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{\left(7.06 \times 10^{-5}\right)^2}}\ \text{m} = 2.05 \times 10^7\ \text{m}$$

(Remember this is the distance from the centre of Mars.)

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