F1

Given that $u_x = 15\,\mathrm{m\,s^{-1}}$, $v_x = 25\,\mathrm{m\,s^{-1}}$ and $t = 9.0\,\mathrm{s}$, where u_x and v_x are the initial and final speeds, respectively, and using the definition of (uniform) acceleration as the rate of change of velocity we obtain

$$a_x = \frac{v_x - u_x}{t} = \frac{(25 - 15)\,\mathrm{m\,s^{-1}}}{9.0\,\mathrm{s}} = 1.1\,\mathrm{m\,s^{-2}}$$

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F2

(a) From the graph (Figure 1), we can see that the parachutist fell for 5 s before opening the parachute.

(b) The acceleration is equal to the gradient of the velocity–time graph, $\Delta v_x/\Delta t$, between 0 s and 5 s, i.e.

$(50 - 0)\,\mathrm{m\,s^{-1}}/(5 - 0)\,\mathrm{s} = 10\,\mathrm{m\,s^{-2}}$.

The acceleration is directed vertically downwards since that is the direction of increasing x in this case. (This is, as might be expected, the acceleration due to gravity.)

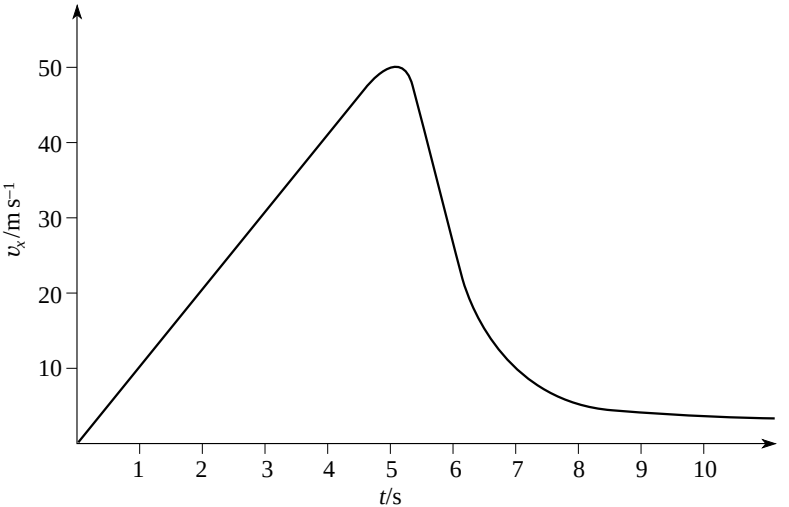

Figure 1 See Question F2.

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(c) The displacement from the start of the descent until the parachute was opened is given by the average velocity multiplied by the time, or by the area under the velocity–time graph between 0 s and 5s.

$$s_x = \langle v_x \rangle \times t = (50\ \mathrm{m\,s^{-1}} \times 5\ \mathrm{s})/2 = 125\ \mathrm{m}$$

Thus, the parachutist fell a distance $|s_x| = 125\ \mathrm{m}$ before opening the parachute.

(d) The acceleration after the parachute opened is given by the gradient of the velocity–time graph for that part of the motion, roughly

$$a_x = \frac{v_{x2} - v_{x1}}{t_2 - t_1} = \frac{(20 - 50)\ \mathrm{m\,s^{-1}}}{(6 - 5)\ \mathrm{s}} = -30\ \mathrm{m\,s^{-2}}$$

Thus, the acceleration during this stage is $30\ \mathrm{m\,s^{-2}}$ vertically upwards.

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F3

A *vector* has magnitude and direction. A *scalar* has no direction. The *magnitude* of a vector is the ‘length’ or ‘size’ of that vector and so $|\boldsymbol{a}|$ can never be negative.

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R1

The *Cartesian coordinates* of A and B are (4, 4) and (2, 8), respectively. The *origin* is the point on the graph where the x and y-*axes* meet.

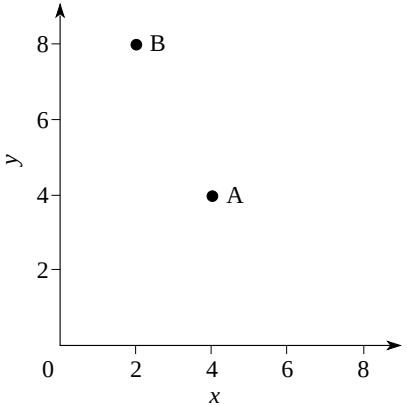

Figure 2 See Question R1.

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R2

The *gradient* of the left-hand graph is approximately
130 pence kg^{-1}. The gradient of the right-hand graph is approximately
$-(2.5/1000)°C\ m^{-1}$
or $-0.0025\ °C\ m^{-1}$
or $-2.5 \times 10^{-3}\ °C\ m^{-1}$.

To find the gradient of any straight line select two convenient points on the line and, between them, measure the horizontal distance in scale units (the *run*) and the vertical distance in scale units (the *rise*), taking due account of signs.

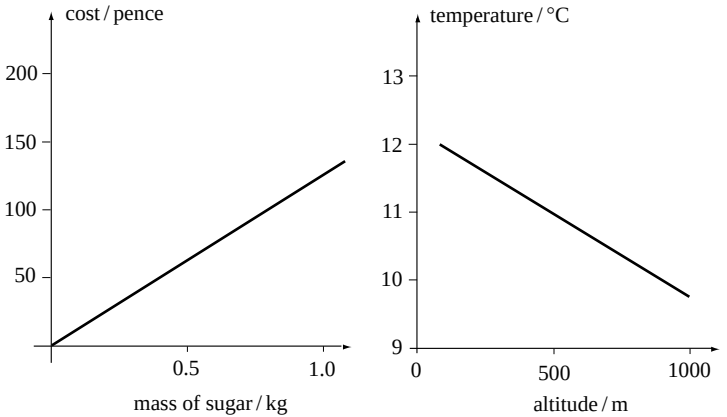

Figure 3 See Question R2.

The gradient of the line is then given by *gradient* = *rise*/*run*.

For further information about *gradients* consult the *Glossary*.

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R3

Mass, kilogram (kg); length, metre (m); time, second (s).

For further information about *SI units*, consult the *Glossary*.

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T1

Your position–time graph from Table 1 should look like Figure 21.

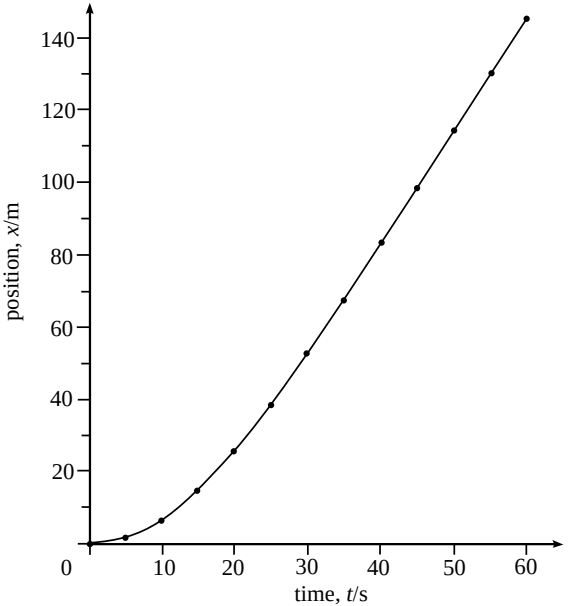

Figure 21 See Answer T1.

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T2

The position after 32 s is approximately 60 m and the time to travel the first 100 m is approximately 45 s.

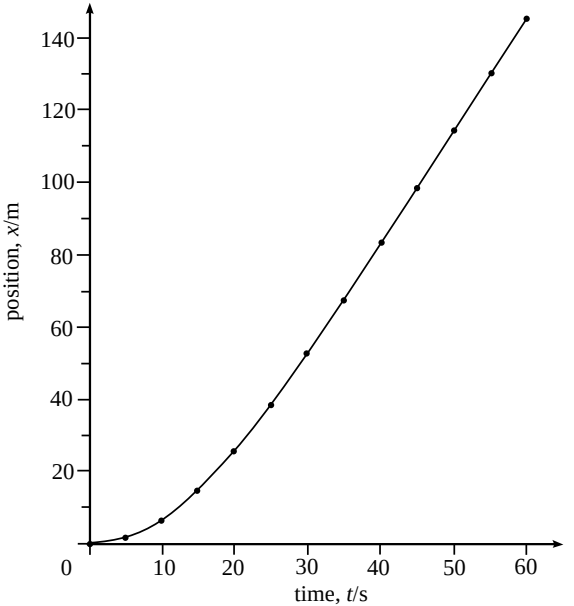

Figure 21 See Answer T1.

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T3

This time the origin is located in the same place as in Figure 10, i.e. at +20 m from the origin of Figure 8, but the *x*-axis now points from right to left. Reversing the direction of increasing *x* turns the position–time graph upside down.

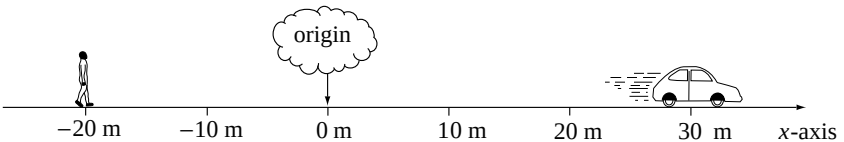

Figure 10 The new coordinate system. (The new origin is displaced (+)20 m from the old origin, i.e. 20 m to the right.)

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T4

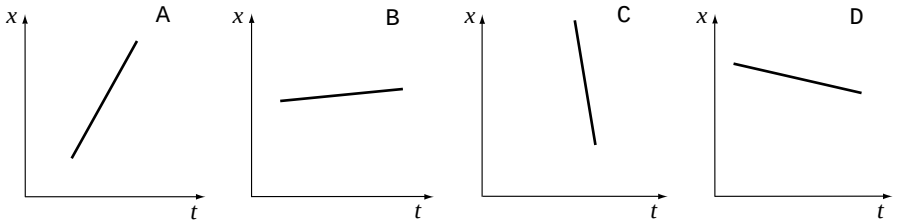

Figure 13 See Question T4.

The steeper the position–time graph, the greater is the change in position in a given interval of time and therefore the greater is the speed. Thus the correct ordering is B, D, A and C. However, only graphs A and B correspond to positive velocities, since these are the only cases in which the position coordinate increases with time. Thus, in order of increasing velocity, the graphs are C, D, B and A. Note that this is the order of increasing *gradients* of the graphs.

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T5

Her average velocity is given by Equation 5.

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad \text{(Eqn 5)}$$

Since x_2 and x_1 are the same (position of the student's home) the change in position $(x_2 - x_1) = 0$ and her average velocity is zero. Her total journey time is 3 h and the total distance covered is 120 miles, so her average speed is the total distance covered divided by time taken, or 120 miles/3 h = 40 mph. Her friend covers the 60 miles, at an average speed of 20 mph, and his journey time is therefore also 3 h, so they arrive at the same time. This question should encourage you to think carefully about *averages* and also about the distinction between *speed* and *velocity*.

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T6

Figure 22 shows the position–time graph for the data in Table 2 with the tangent lines drawn.

$$\text{Gradient at } 0.20\,\text{s} = \frac{(1.4-0)\,\text{m}}{(0.80-0.11)\,\text{s}} = \frac{1.4\,\text{m}}{0.69\,\text{s}}$$

$$= 2.0\,\text{m}\,\text{s}^{-1}$$

$$\text{Gradient at } 0.60\,\text{s} = \frac{(2.9-0)\,\text{m}}{(0.80-0.30)\,\text{s}} = \frac{2.9\,\text{m}}{0.50\,\text{s}}$$

$$= 5.8\,\text{m}\,\text{s}^{-1}$$

You may not necessarily have chosen the same time intervals for the gradient calculations, but your answers should be approximately the same as those above, given the limitations of the scale.

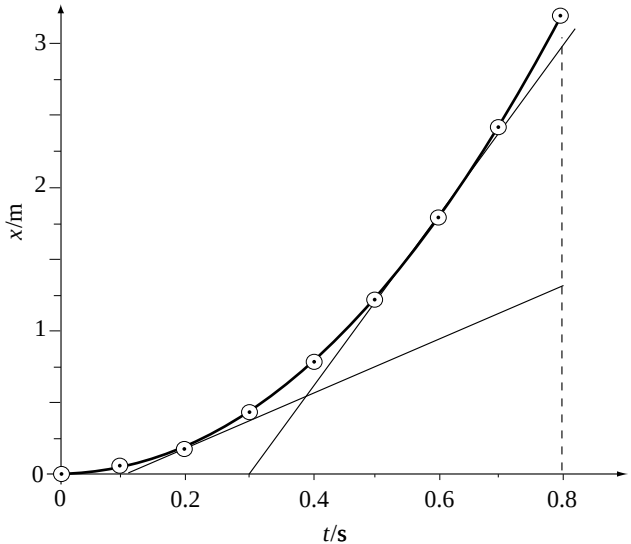

Figure 22 See Answer T6.

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The average velocity between $t = 0.10\,\text{s}$ and $0.30\,\text{s}$ can be obtained using Equation 5 and the data directly from Table 2

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad \text{(Eqn 5)}$$

$$\langle v_x \rangle = \frac{(0.44 - 0.05)\,\text{m}}{(0.30 - 0.10)\,\text{s}} = \frac{0.39}{0.20}\,\text{m s}^{-1} = 2.0\,\text{m s}^{-1}$$

and between $t = 0.50\,\text{s}$ and $0.70\,\text{s}$

$$\langle v_x \rangle = \frac{(2.40 - 1.24)\,\text{m}}{(0.70 - 0.50)\,\text{s}} = \frac{1.16}{0.20}\,\text{m s}^{-1} = 5.8\,\text{m s}^{-1}$$

We see that these average velocities are the same as our instantaneous velocities at $t = 0.20\,\text{s}$ and $t = 0.60\,\text{s}$, to within the accuracy of our graph.

Table 2 See Question T6.

Time t/s	Position x/m
0.00	0.00
0.10	0.05
0.20	0.20
0.30	0.44
0.40	0.78
0.50	1.24
0.60	1.75
0.70	2.40
0.80	3.15

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T7

Your velocity–time graph should look like Figure 23.

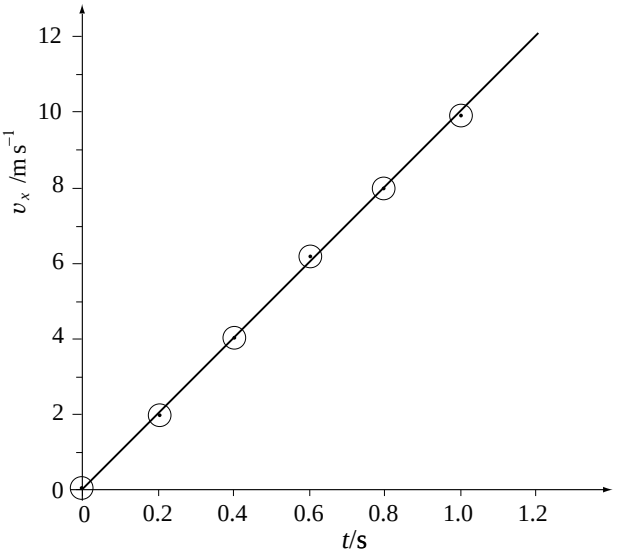

Figure 23 See Answer T7.

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T8

If we take the interval between 0 s and 1.0 s the velocity increases from $0\,\mathrm{m\,s^{-1}}$ to $10\,\mathrm{m\,s^{-1}}$. Therefore the gradient is approximately $10\,\mathrm{m\,s^{-2}}$.

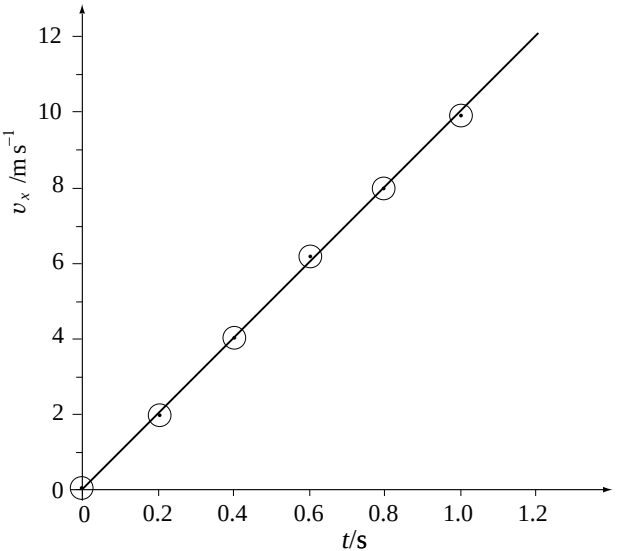

Figure 23 See Answer T7.

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T9

From measurements on the tangents to the curve at the three times the three gradients are approximately $0.66\ \mathrm{m\,s^{-2}}$, $0\ \mathrm{m\,s^{-2}}$, $-1.3\ \mathrm{m\,s^{-2}}$, respectively. These are therefore the values of the accelerations at these times.

The journey can be described as follows. The car accelerates from rest to a steady velocity of $17.5\ \mathrm{m\,s^{-1}}$ in about 34 s. It remains at that velocity for about 10 s and then decelerates to rest in about 15 s, with the deceleration increasing as the velocity is reduced to zero — this looks like an emergency stop!

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T10

Between t_1 and t_2 the area under the graph is

$$u_x \times (t_2 - t_1) = u_x t_2 - u_x t_1$$

but, if s_{x1} is the displacement at time t_1 and s_{x2} is the displacement at time t_2, it follows from Equation 8 $\quad s_x = u_x t$

that $\quad u_x t_2 - u_x t_1 = s_{x2} - s_{x1}$

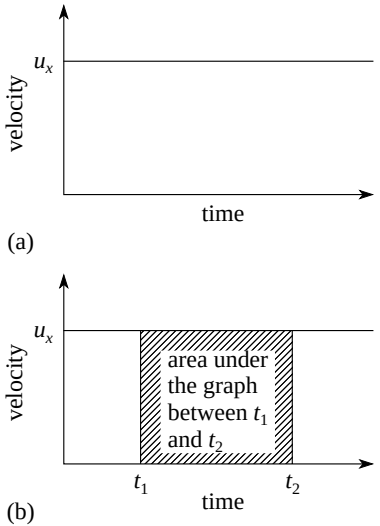

Figure 19 (a) Velocity–time graph of a particle moving with a constant velocity u_x. (b) The area under the velocity–time graph between t_1 and t_2. Note that if the velocity u_x is negative then the area 'under' the graph is also counted as negative.

and the right-hand side of this equation is just the change in displacement, as claimed.

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T11

For the first part of the question $u_x = 0\,\mathrm{m\,s^{-1}}$ and $a_x = 9.0\,\mathrm{m\,s^{-2}}$. From Equation 11

$$v_x = u_x + a_x t \qquad \text{(Eqn 11)}$$

$$v_x = 0\,\mathrm{m\,s^{-1}} + (9.0\,\mathrm{m\,s^{-2}} \times 2.5\,\mathrm{s}) = 22.5\,\mathrm{m\,s^{-1}}$$

This gives a final velocity of $23\,\mathrm{m\,s^{-1}}$ to two significant figures.

For the second part, $u_x = 30\,\mathrm{m\,s^{-1}}$ and $v_x = 40\,\mathrm{m\,s^{-1}}$. To find t, rearrange Equation 11 to get

$$t = \frac{v_x - u_x}{a_x} = \frac{(40 - 30)\,\mathrm{m\,s^{-1}}}{9.0\,\mathrm{m\,s^{-2}}} = 1.1\,\mathrm{s}$$

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T12

Given that $u_x = 0\,\mathrm{m\,s^{-1}}$, $a_x = 9.8\,\mathrm{m\,s^{-2}}$ and $t = 2.0\,\mathrm{s}$, if we use Equation 12

$$s_x = \frac{u_x + v_x}{2}\,t \qquad \text{(Eqn 12)}$$

we find

$$s_x = 0\,\mathrm{m} + \frac{1}{2}\left[9.8\,\mathrm{m\,s^{-2}} \times (2.0\,\mathrm{s})^2\right] = 19.6\,\mathrm{m}$$

So after 2.0 s the stone will have fallen a distance $|s_x| = 20\,\mathrm{m}$, to two significant figures.

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T13

Given that $s_x = 2.0\,\text{m}$, $a_x = g = 9.8\,\text{m s}^{-2}$ and $u_x = 0\,\text{m s}^{-1}$, if we use Equation 11 to relate v_x to t

$$v_x = u_x + a_x t \qquad \text{(Eqn 11)}$$

we find

$$v_x = 0\,\text{m s}^{-1} + gt$$

i.e. $\quad t = v_x / g$

We do not know the time, t, but Equation 13

$$s_x = u_x t + \tfrac{1}{2} a_x t^2 \qquad \text{(Eqn 13)}$$

also involves t. If we substitute the above values into Equation 13 we find

$$2.0\,\text{m} = 0\,\text{m} + \frac{1}{2} g t^2$$

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If we then substitute for t we get

$$2.0\,\mathrm{m} = \frac{1}{2}g\left(\frac{v_x}{g}\right)^2 = \frac{1}{2}g\frac{{v_x}^2}{g^2} = \frac{{v_x}^2}{2g}$$

so $\quad {v_x}^2 = 4.0\,\mathrm{m} \times g = 39.2\,\mathrm{m}^2\,\mathrm{s}^{-2}$

Taking the positive square root gives $v_x = 6.26\,\mathrm{m\,s^{-1}}$.

Therefore the stone's final velocity is $6.3\,\mathrm{m\,s^{-1}}$, to two significant figures.

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T14

Given that $u_x = 6.0\,\mathrm{m\,s^{-1}}$, $a_x = 3.0\,\mathrm{m\,s^{-2}}$ and $s_x = 20\,\mathrm{m}$, the final velocity, v_x, can be found from Equation 14

$$v_x^2 = u_x^2 + 2a_x s_x \qquad \text{(Eqn 14)}$$

$$v_x^2 = (6.0\,\mathrm{m\,s^{-1}})^2 + (2 \times 3.0\,\mathrm{m\,s^{-2}} \times 20\,\mathrm{m})$$

$$= 156\,\mathrm{m^2\,s^{-2}}$$

Taking the positive square root gives $v_x = 12.5\,\mathrm{m\,s^{-1}}$.

The car's final velocity is therefore $13\,\mathrm{m\,s^{-1}}$, to two significant figures.

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