Boltzmann's constant
$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$.

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We could, if we preferred, regard the first set as the set of all quotients T/V formed from pairs of values for V and T but that would hardly be an improvement.

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Note that $f(x)$ does *not* mean $f \times x$.

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Computers often make use of tables of values to define functions, though computer scientists call them ‘look-up tables’.

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Note that the displacements referred to here are the ‘scale’ displacements shown on the axes.

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It would not be correct to draw a curve through the points on the graph in Figure 2 as it is the graph of an equation with discrete variables (see Subsection 2.4).

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Throughout the rest of this section we will always assume that x may be any real number unless stated otherwise.

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We can call this the ‘etiquette’ of graph drawing, because it is good manners to make things easy for anyone you are communicating with — quite apart from the fact that it is always to your advantage to get your points over as clearly as possible.

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Computer graphics programs, which plot graphs automatically, are useful but not usually very intelligent. They often produce unsuitable scales.

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In bygone days you might have been advised to indicate units by brackets as in mass (kg). This is now regarded as potentially confusing, so you should use the mass/kg notation instead.

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You should note, however, that graph paper has not been used for the graphs in *FLAP* although a coarse grid has been included. This is purely a technical compromise as the modules were designed to be photocopied.

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The gradient of a line is sometimes called its *slope*.

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If you are confused by this answer just replace all occurrences of x^2 by some other symbol such as z.

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Note that Δx is a *single* quantity and *not* the product of some quantity Δ and some value of x.

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Remember, that all these separations are to be measured in the scale units appropriate to the graph. Thus the units of m (if there are any) will be the same as the units of the ratio y/x.

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$h(x)$ may not look like a quadratic function, but if you expand the brackets you will see that it is

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$h(x) = x^2 + 4x + 4$

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Maxima, minima and turning points are defined more precisely and discussed more extensively in the modules dealing with differentiation. See the *Glossary* for details.

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This question is quite challenging, try to work it out for yourself but if you get stuck look at the answer.

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Expand the inner bracket to confirm that this is correct.

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In *FLAP* roots are generally treated as positive or negative quantities, so strictly speaking the $\pm$ is not needed here. However, to omit it would be highly unconventional so we prefer to regard it as a *reminder* that the root may be positive or negative.

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You can check this by substituting either of the values for x given in Equation 17 into Equation 18.

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The constant coefficients have been denoted by the subscripted symbols a_0, a_1, a_2, etc. to make it easier to associate them with the appropriate powers of x.

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You may well have used different symbols for the variables and constants.

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The symbol $\gg$ should be read as ‘much greater than’, and $\ll$ should be read as ‘much less than’.

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A more precise definition of a point of inflection is given in one of the modules dealing with differentiation — see the *Glossary* for details.

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The word ‘local’ has been inserted to stress that these turning points are not really the maximum and minimum values of the function as a whole.

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The symbol $\neq$ should be read as ‘is not equal to’.

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The symbol ≈ should be read as ‘is approximately equal to’.

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Some authors indicate the inverse function of $F(x)$ by $F^{-1}(x)$. Due to danger of misinterpreting this as $1/F(x)$ we prefer to use a totally different symbol such as $G(x)$.

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Of course, the domains of $g(y)$ and $g(x)$ also have to be the same.

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The subscript + and – signs in these examples identify $f_+(x)$ and $f_-(x)$ as different functions since their definitions differ in sign — however, note they have the same domain. We could have called them $F(x)$ and $G(x)$.

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✧

Yes. There is only one sunrise time for each day (this assumes that you don't live within the Arctic or Antarctic Circle). ❑

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✧

The rule is obvious, it's the ideal gas equation $PV = NkT$. The second of the two sets is equally obvious, it's the set of all possible values for the pressure. The difficult item to identify is the first set: it can't be the set of all possible temperatures since the volume may vary, so a given temperature is no longer associated with a single pressure; nor, since the temperature may vary, can it be the set of all possible volumes. Instead, the first set is the set of all possible *pairs* of values for the volume and temperature taken together. The set of all possible pairs of values (V, T) may seem like a rather strange set to have to consider, but it's the price we pay for regarding P as a function of V and T in the given sample. ❑

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✧

The equation indicates that the dependent variable V is a function of the independent variables I and R. In other words, a given pair of values for I and R determine a single value of V. (We have followed the physicist's convention of representing the function by V, even though this symbol has already been used to represent the dependent variable. A mathematician would probably have written the same relationship as $V = f(I, R)$). ❑

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✧

$n = m^2 + m + 1$. This equation is the result of restricting the values of the real variable x in the equation $y = x^2 + x + 1$ (given above) to the given integer values of m (i.e. $1 \leq m \leq 9$). ❑

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✧

(a) $f(x)$ is linear, it is equal to $2x + 1$;
$m = 2$, $c = 1$.

(b) $f(t)$ is linear; $m = -6.0$,
$c = -3.8 \times 10^4$.

(c) $f(x)$ is linear; $m = 6$, $c = a$
(You may not know the value of a, but that makes no difference as long as you know it's constant.)

(d) $f(x)$ is linear; in this case the gradient is c and the intercept is b.
Don't be confused by the fact that the given function includes a constant called c. That constant has no connection whatsoever with the symbol c that is conventionally used to represent the intercept.

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(e) $f(x)$ is *not* linear, so the terms gradient and intercept do not apply.

(f) $f(x^2)$ is linear, but note that it is a linear function of x^2 (though *not* a linear function of x). In other words if we let $y = f(x^2)$ and plot the graph of y against x^2 the result will be a straight line (though a graph of y against x would *not* be straight). Regarded as a function of x^2, the gradient is $m = 3$ and the intercept $c = -2.1$. ❑

✧

(a) To find the gradient:

1 Pick two convenient points, (x_1, y_1) and (x_2, y_2), on the straight line.

2 Determine their horizontal separation $x_2 - x_1$. (This corresponds to the separation h in Figure 5a, and is often called the *run*.) ☞

3 Determine their vertical separation $y_2 - y_1$, taking the points in the same order as before. (This corresponds to the separation mh in Figure 5a and is often called the *rise*.)

4 Then, from Equation 6,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

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(b) To find the intercept:

Simply find the value of y that corresponds to $x = 0$. (*Note* if your graph does not include $x = 0$, do not make the mistake of taking the value of y at which the line crosses your vertical axis to be the intercept. It must be the y value at $x = 0$.) ❑

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✧

For $h(x)$; $a = 1$, $b = 4$, $c = 4$.

For $j(x)$; $a = -1$, $b = 0$, $c = -3$. ❑

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✧

$f(x) = a + bx + cx^2 + dx^3$
is a cubic function

$f(x) = a + bx + cx^2 + dx^3 + ex^4$
is a quartic function.

✧

When $x = -2$, $y = f(-2) = 2(-2) + 1 = -3$,

and when $y = -3$, $g(-3) = (-3 - 1)/2 = -2$.

When $x = 0$, $y = f(0) = 2(0) + 1 = 1$,

and when $y = 1$, $g(1) = (1 - 1)/2 = 0$.

When $x = 3$, $y = f(3) = 2(3) + 1 = 7$,

and when $y = 7$, $g(7) = (7 - 1)/2 = 3$.

So, for these three values of x at least, the functions $f(x)$ and $g(y)$ satisfy the requirement that $g(y)$ should 'undo' the effect of $f(x)$. ❑

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