E1

The graph of the function $f(x)$ is given in Figure 20.

In this case there is a definite kink at the origin. The function can be defined as $f(x) = |x|$.

(Reread Subsections 2.4, 3.1 and 3.2 if you had difficulty with this question.)

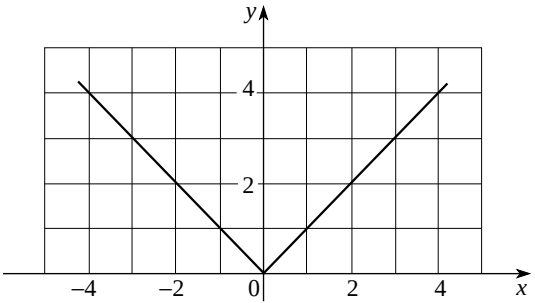

Figure 20 See Answer E1.

COPYRIGHT © 1998 THE OPEN UNIVERSITY S570 V1.1

E2

See the sketches in Figure 21
(Reread Subsections 3.1, 3.2, 3.3 and 3.4 if you had difficulty with this question.)

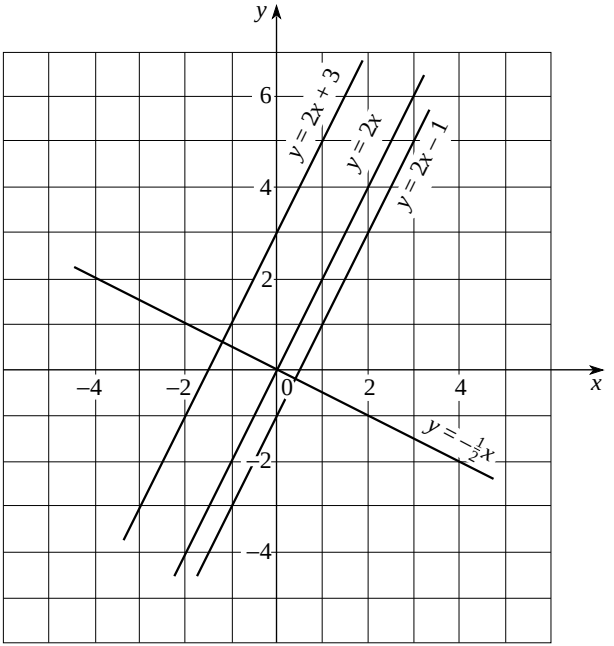

Figure 21 See Answer E2.

E3

Using the point–gradient form of the equation of a straight line,

$$(y - 5)/(x - 3) = 3; \quad \text{so } y - 5 = 3(x - 3), \quad \text{i.e. } y = 3x - 4$$

(Reread Subsection 3.4 if you had difficulty with this question.)

COPYRIGHT © 1998 THE OPEN UNIVERSITY S570 V1.1

E4

Using the two-point form of the equation of a straight line,

$$(y - 5)/(x + 1) = (-1 - 5)/(2 + 1) = -6/3 = -2;$$

$$\text{so } y = -2x + 3$$

(Reread Subsection 3.4 if you had difficulty with this question.)

COPYRIGHT © 1998 THE OPEN UNIVERSITY S570 V1.1

E5

$f(x)$ is in *completed square form* (Equation 14) and has its vertex at (0, –3); $g(x)$ is also in completed square form and its vertex is at (0, 1); $h(x)$ may be written in completed square form as $h(x) = (x + 1)^2 + 1$ and has its vertex at (–1, 1).

$f(x)$ and $g(x)$ both intersect the x-axis twice since they both have *discriminants* that are greater than zero. $h(x)$ has a negative discriminant, so it does not cut the axis. For $f(x)$ the points of intersection are $x = \pm\sqrt{6}$, and for $g(x)$ they are $x = \pm\sqrt{2}$. (These values may be obtained from Equation 17, but in this case it is easier to deduce them directly from the given functions.)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(Eqn 17)}$$

The graphs are shown in Figure 22.

(Reread Subsection 3.5 if you had difficulty with this question.)

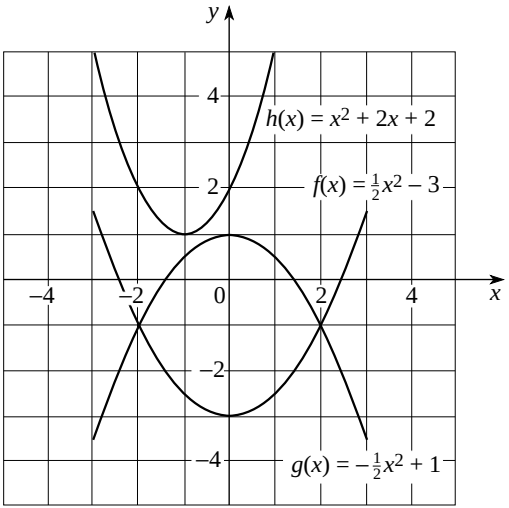

Figure 22 See Answer E5.

E6

The sketch of $y = (x - 1)^3$ is given in Figure 23. The curve $y = (x - 1)^3$ is the same shape as the curve in Figure 8, but it has been displaced along the x-axis by 1 scale unit to the right. (If we had let $(x - 1) = X$ we would have been plotting $y = X^3$.) The curve crosses the x-axis once. Its point of inflection is at (1, 0).

The curve $y = x^3 - 9x$ is similar to that of Figure 9, it crosses the x-axis three times and has a point of inflection at the origin. (The fact that the point of inflection is close to the origin in this case should be obvious from your graph. However, the proof that this is its exact location is beyond the scope of this module.)

(Reread Subsection 3.6 if you had difficulty with this question.)

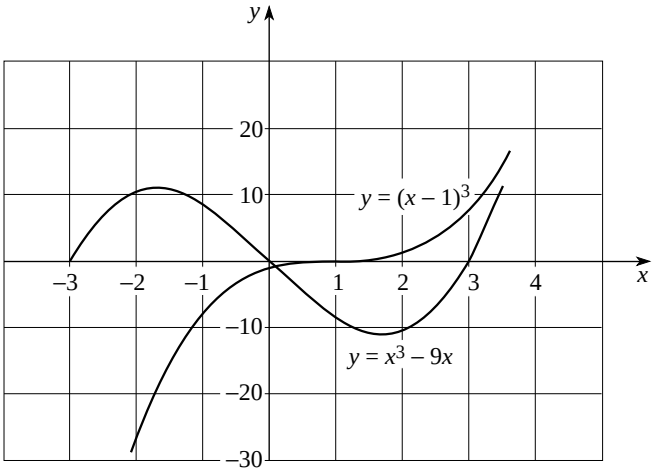

Figure 23 See Answer E6.

E7

See Figure 24. (Note the inclusion of SI units in the axis labels.)

After the first two seconds the velocity becomes *negative*, i.e. the stone is moving downwards.

(Reread Subsection 3.3 and 3.4 if you had difficulty with this question.)

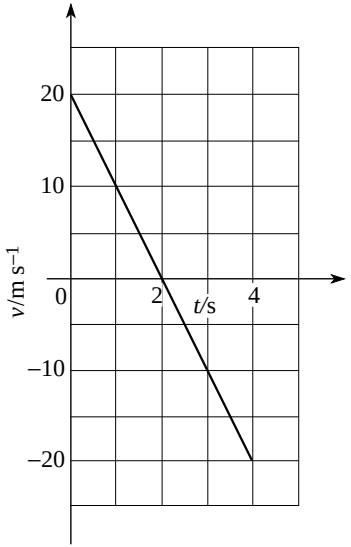

Figure 24 See Answer E7.

E8

The graph of the function is given in Figure 25.

(a) The maximum height (determined by the vertex of the parabola) is 20 m.

(b) The total time (determined by the difference in the t values at the two points of intersection with the t-axis) is 4 s.

(Reread Subsection 3.5 if you had difficulty with this question.)

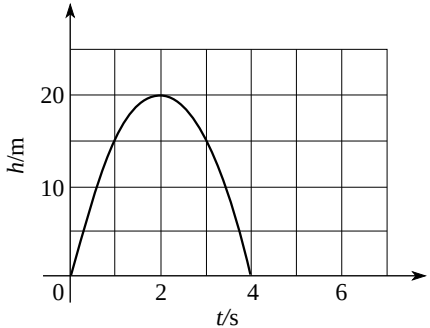

Figure 25 See Answer E8.

COPYRIGHT © 1998 THE OPEN UNIVERSITY S570 V1.1

E9

The graph of the function is given in Figure 26. As x becomes larger and larger (either positively or negatively), the expression under the square root symbol gets closer and closer to x^2, so, $f(x) \approx |x|$ for large values of x. You determined the shape of this function in Question E1, thus the asymptotes are the lines $y = x$ and $y = -x$, and the curve approaches each of them from *above*. The function $f(x)$ does not have an inverse because a particular value of $f(x)$ may correspond to several different values of x.

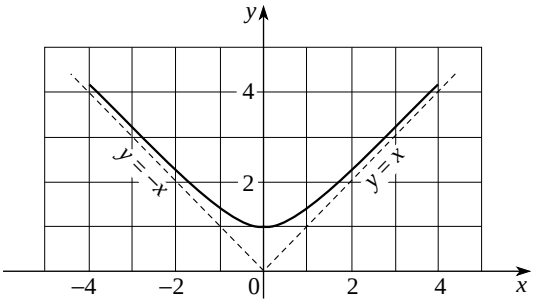

Figure 26 See Answer E9.

(Reread Subsections 3.7 and 4.1 if you had difficulty with this question.)

COPYRIGHT © 1998 THE OPEN UNIVERSITY S570 V1.1

E10

$f(g(y)) = f(y + 1) = y^2$, where y may be any real number

$g(f(y)) = g((y - 1)^2) = (y - 1)^2 + 1 = y^2 - 2y + 2$, where y may be any real number.

(Reread Subsections 4.2 and 2.3 if you had difficulty with this question.)

COPYRIGHT © 1998 THE OPEN UNIVERSITY S570 V1.1