F1

(a) $z = [4 - (1 + 2)^2]^2 = (4 - 9)^2 = (-5)^2 = 25$

(b) $z = 4x^2y^2 - 4xy(x + y)^2 + (x + y)^4$

$$= 4x^2y^2 - 4xy(x^2 + 2xy + y^2) + (x^2 + 2xy + y^2)(x^2 + 2xy + y^2)$$

$$= 4x^2y^2 - 4x^3y - 8x^2y^2 - 4xy^3 + x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$= x^4 + 2x^2y^2 + y^4$$

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F2

(a) $$2mu - \frac{2}{u}d = \frac{2mu^2 - 2d}{u} = \frac{2\left(mu^2 - d\right)}{u}$$

(b) $$\frac{2l^2}{3h} + \frac{h}{2} = \frac{4l^2 + 3h^2}{6h}$$

(c) $$\frac{3l(h+f)^2}{2a^2} \div \frac{3a\left(h^2 - f^2\right)}{-2l^2}$$

$$= \frac{3l(h+f)^2}{2a^2} \times \frac{\left(-2l^2\right)}{3a\left(h^2 - f^2\right)} = \frac{-6l^3(h+f)^2}{6a^3(h+f)(h-f)}$$

$$= \frac{-l^3(h+f)}{a^3(h-f)} = -(l/a)^3 \frac{(h+f)}{(h-f)}$$

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F3

The value of v that will just permit escape is given by $m_1v^2/2 = Gm_1m_2/r$. Cancelling a common factor of m_1 on both sides and multiplying by 2 gives $v^2 = 2Gm_2/r$.

Taking the positive square root gives $v = \sqrt{2Gm_2/r}$.

The inequality $v > \sqrt{2Gm_2/r}$ gives the condition for escape.

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T1

(a) $3 + 6/2 - 4 = 3 + 3 - 4 = 2$

(b) $4.2 \times 3.0 - 1.4 = 12.6 - 1.4 = 11.2$

(c) $ab + c/2 = 2 \times 1/2 + (-4)/2 = 1 - 2 = -1$

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T2

(a) $(6 + 3/(9 - 7))/2 = (6 + 3/2)/2$

$= (6 + 1.5)/2 = 7.5/2 = 3.75$

(b) $2\,[(3a - 4)b + 2a(3b + 1)] = 2[(3 - 4)(-1) + 2\,(-3 + 1)]$

$= 2[(-1)(-1) + 2(-2)] = 2(1 - 4) = -6.$

(c) $2\{[2 - 2(2 + 3)/(2 - 1)] + [1 + (2 + 3)/2]/2\}$

$= 2[(2 - 10) + (1 + 2.5)/2] = 2\,(-8 + 1.75) = -12.5$

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T3

(a) $2 + 4/3^2 = 2 + 4/9 = 2 + 0.44 = 2.44$

(b) $2 + (4 - 3a)^2 = 2 + (4 - 3)^2 = 2 + 1 = 3$

(c) $(2 + (2 + x)^2)^2 = (2 + (2 + 2)^2)^2 = (2 + 4^2)^2 = (2 + 16)^2 = 18^2 = 324$

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T4

(a) $3x(4 + 3y) + 2x = 12x + 9xy + 2x = 14x + 9xy$

(b) $(x + 2y)^2 = (x + 2y)(x + 2y) = x^2 + 2xy + 2yx + 4y^2 = x^2 + 4xy + 4y^2$

(c) $(x + y)(x - y) = x^2 - xy + yx - y^2 = x^2 - y^2$

(d) $(3x + 6y)/2 = 3x/2 + 6y/2 = 3x/2 + 3y$

(e) $(p + q)/q = p/q + q/q = p/q + 1$

(f) $(2xy)^2/2 = 4x^2y^2/2 = 2x^2y^2$

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T5

(a) $3x^2y + 6xy = 3xy(x + 2)$

(b) $8x^3 + 4x^2y + 2xy^2 + 2x = 2x(4x^2 + 2xy + y^2 + 1)$

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T6

(a) $7/42 = 7/(6 \times 7) = 1/6$

(b) $6x/(2x^2) = (3 \times 2x)/(2x \times x) = 3/x$

(c) $8(x + 3)/(2x) = 4(x + 3)/x$

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T7

(a) The denominator is $(3x^2y + 6xy) = 3xy(x + 2)$, so the numerator and denominator have a common factor $(x + 2)$ and the fraction becomes $(x + 3)/(3xy)$.

(b) The numerator is $(3xy + 3y^2) = 3y(x + y)$, and the denominator is $(4x^2y + 4x^3) = 4x^2(y + x)$, so cancelling a common factor of $(x + y)$ in the fraction gives $3y/4x^2$.

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T8

(a) $\frac{2}{3}+\frac{3}{5}=\frac{10}{15}+\frac{9}{15}=\frac{19}{15}$

(b) $\frac{x}{3}+\frac{x}{2}=\frac{2x}{6}+\frac{3x}{6}=\frac{5x}{6}$

(c) $\frac{1}{u}+\frac{1}{v}=\frac{v}{uv}+\frac{u}{uv}=\frac{v+u}{uv}$

(d) $\frac{1+a}{3+a}$ cannot be simplified

(e) $\frac{2(1+x)}{(1+x)^2}=\frac{2}{(1+x)}$

(f) $\frac{5/7}{2/3}=\frac{5}{7}\times\frac{3}{2}=\frac{15}{14}$

(g) $\frac{3x}{x/3}=3x\times\frac{3}{x}=\frac{9x}{x}=9$

(h) $\frac{3}{a}+\frac{7}{2a}=\frac{6}{2a}+\frac{7}{2a}=\frac{13}{2a}$

(i) $\left(-7x^2\right)\div(-3x)=\frac{7x^2}{3x}=\frac{7x}{3}$

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T9

(a) $2^2 \times 2^3 = 2^5 = 32$

(b) $2^3 \times 2^4 \times 2^5 = 2^{12} = 4\,096$

(c) $(-2)^4 \times (-2)^2 = (-2)^6 = 64$

(d) $(2^3)^2 = 2^6 = 64$

(e) $[(-0.2)^2]^3 = (-0.2)^6 = 64 \times 10^{-6} = 6.4 \times 10^{-5}$

(f) $[(-1)^{17}]^{23} = (-1)^{17 \times 23} = (-1)^{391} = -1$

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T10

(a) $2^{-4} = 1/2^4 = 1/16 = 0.062\,5$

(b) $10^{-3} = 1/10^3 = 0.001$

(c) $\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$

(d) $(0.2)^{-2} = 1/(0.2)^2 = 1/0.04 = 25$

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T11

(a) $5^2 \times 5^{-5} = 5^{2-5} = 5^{-3}$

(b) $(5^2)^{-3} = 5^{-6}$

(c) $2^4/2^6 = 2^4 \times 2^{-6} = 2^{4-6} = 2^{-2}$

(d) $(3^{-1})^{-1} = 3^1$ (i.e. 3)

(e) $\left(\frac{1}{2}\right)^{-3}$ is already in the required form, with $a = \frac{1}{2}$. However, it may be rewritten as $(2^{-1})^{-3} = 2^3$ if desired. Also 8^1 is correct.

(f) $$\left(\frac{2^5}{2^6}\right)^{-1} \times 2 \times 2^3 = \left(\frac{2^6}{2^5}\right) \times 2 \times 2^3 = \frac{2^{10}}{2^5} = 2^{10-5} = 2^5$$

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T12

(a) $9^{1/2} = 3$ or -3 (often written $9^{1/2} = \pm 3$)

(b) $16^{1/2} = 4$ or -4 (often written $16^{1/2} = \pm 4$)

(c) $16^{1/4} = 2$ or -2 (often written $16^{1/4} = \pm 2$)

(d) $27^{1/3} = 3$

(e) $\sqrt{64} = 8$ or -8 (often written $\sqrt{64} = \pm 8$)

(f) $\sqrt[4]{49} = \sqrt{7} = 2.65$ or -2.65 (often written $\sqrt[4]{49} = \pm 2.65$)

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T13

(a) $(-8)^{1/3} = -2$

(b) $\left(-\dfrac{1}{16}\right)^{1/2}$ cannot be found since it is the square root of a negative number.

(c) $(-2)^{1/2}$ cannot be found. It too is the square root of a negative number.

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T14

(a) $7^{4/9} = 2.37$

(b) $(2.7)^3/(2.7)^2 = (2.7)^{3-2} = (2.7)^1 = 2.7$

(c) $\sqrt{2} \times \sqrt[3]{2} = 2^{1/2} \times \left(2^{1/3}\right) = 2^{1/2+1/3} = 2^{5/6} = 1.78$

(d) $\dfrac{\sqrt{3}}{\sqrt[5]{3}} = 3^{1/2} \times \left(3^{1/5}\right)^{-1} = 3^{1/2-1/5} = 3^{3/10} = 1.39$

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T15

(a) $1/\sqrt{r} = 1/r^{1/2} = r^{-1/2}$

(b) $r^3\sqrt{r} = r^3 \times r^{1/2} = r^{7/2}$

(c) $r^5 / \sqrt{r} = r^5 / r^{1/2} = r^5 \times r^{-1/2} = r^{9/2}$

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T16

(a) $4x^2y^2z^4 = (2xyz^2)^2$

(b) $$\frac{h^2n^2}{8m}\left(\sqrt[3]{L^3}\right)^{-2} = \frac{h^2n^2L^{-2}}{8m} = \frac{1}{8m}\left(\frac{hn}{L}\right)^2$$

(c) $$\left(\frac{8\pi^2 Z\sqrt{l^5}}{3\sqrt[3]{Z}}\right)^{1/2} = \left(\frac{8\pi^2 Z^{2/3} l^{5/2}}{3}\right)^{1/2}$$

$$= \frac{2\sqrt{2}\pi Z^{2/6} l^{5/4}}{\sqrt{3}} = \left(\frac{8}{3}\right)^{1/2} \pi Z^{1/3} l^{5/4}$$

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T17

If a common factor of $\dfrac{\theta}{b^2 - a^2}$ is extracted from each term on the right-hand side

$$\phi = \left(b^2 Vr - \frac{b^2 Va^2}{r} - a^2 Ur + \frac{a^2 Ub^2}{r} \right) \frac{\theta}{b^2 - a^2}$$

If a common factor of b^2V is extracted from the first two terms and a common factor of $-a^2U$ from the second two terms

$$\phi = \left[b^2 V \left(r - \frac{a^2}{r} \right) - a^2 U \left(r - \frac{b^2}{r} \right) \right] \frac{\theta}{b^2 - a^2}$$

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T18

$$F = 2\pi r\left[\frac{2}{3}r^2 g(\rho - \sigma) - 3\eta v\right]$$

So, when $F = 0$ and $v = v_t$ we have

$$0 = \frac{2}{3}r^2 g(\rho - \sigma) - 3\eta v_t$$

If $3\eta v_t$ is added to both sides

$$3\eta v_t = \frac{2}{3}r^2 g(\rho - \sigma) - 3\eta v_t + 3\eta v_t$$

The last two terms on the right-hand side cancel, so if both sides are divided by 3η

$$v_t = \frac{2r^2}{9\eta}g(\rho - \sigma)$$

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T19

(a) $\dfrac{F_\mathrm{b}}{F_\mathrm{g}} = \dfrac{(4/3)\pi r^3 \sigma g}{(4/3)\pi r^3 \rho g} = \dfrac{\sigma}{\rho}$

$$\frac{F_\mathrm{v}}{F_\mathrm{g}} = \frac{6\pi\eta r v}{(4/3)\pi r^3 \rho g} = \frac{3\eta v}{2r^2 \rho g/3} = \frac{9\eta v}{2r^2 \rho g}$$

$$\frac{F_\mathrm{b}}{F_\mathrm{v}} = \frac{(4/3)\pi r^3 \sigma g}{6\pi\eta r v} = \frac{2r^2 \sigma g}{9\eta v}$$

(b) If $\dfrac{F_\mathrm{v}}{F_\mathrm{b}} = 10$ when $v = v_\mathrm{t}$

$$10 = \frac{F_\mathrm{v}}{F_\mathrm{b}} = 1\big/\left(F_\mathrm{b}/F_\mathrm{v}\right) = \frac{9\eta v_\mathrm{t}}{2r^2 \sigma g}$$

If we multiply both sides by $\dfrac{r^2}{10}$, $\quad r^2 = \dfrac{9\eta v_\mathrm{t}}{20\sigma g}$

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Since r^2 is positive we can take square roots of both sides, and since we know that r itself (a radius) is also positive we can neglect negative roots. Thus, $r = \sqrt{\dfrac{9\eta v_t}{20\sigma g}}$

(c) If $F_g = F_v$ $\left(\text{i.e.} \dfrac{F_v}{F_g} = 1\right)$ when $v = v_t$ $\quad 1 = \dfrac{9\eta v_t}{2r^2\rho g}$

If both sides are multiplied by r^2, $\quad r^2 = \dfrac{9\eta v_t}{2\rho g}$

If positive square roots are taken as before $\quad r = \sqrt{\dfrac{9\eta v_t}{2\rho g}}$

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T20

(a) If $y = 2ax + b^2$

then $2ax = y - b^2$ so $x = \dfrac{y - b^2}{2a}$ provided $a \neq 0$

(b) If $(y - b) + (x - a) = 5xh^2 + 3$

then $(x - a) - 5xh^2 = 3 - (y - b)$

so $x - 5xh^2 = 3 - (y - b) + a$

i.e. $x(1 - 5h^2) = 3 - y + b + a$

If both sides are divided by $(1 - 5h^2)$

$$x = \frac{3 - y + a + b}{\left(1 - 5h^2\right)} \quad \text{provided } (1 - 5h^2) \neq 0$$

(The condition $(1 - 5h^2) \neq 0$ is required to make sure we have not unwittingly divided by zero.)

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(c) If $\dfrac{1}{x-a}+\dfrac{1}{y-b}=3t^2$ then $\dfrac{1}{x-a}=3t^2-\dfrac{1}{y-b}$

so $\dfrac{1}{x-a}=\dfrac{3t^2(y-b)-1}{y-b}$

If both sides are multiplied by $\dfrac{(x-a)(y-b)}{3t^2(y-b)-1}$

$$\frac{y-b}{3t^2(y-b)-1}=x-a \quad \text{provided } 3t^2(y-b)-1\neq 0$$

thus $x=\dfrac{y-b}{3t^2(y-b)-1}+a$ provided $3t^2(y-b)-1\neq 0$

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(d) If $t + a = \sqrt{\dfrac{3b}{(x-y)}}$ then $(t + a)^2 = \dfrac{3b}{x-y}$

If both sides are multiplied by $\dfrac{x-y}{(t+a)^2}$

$$x - y = \frac{3b}{(t+a)^2} \quad \text{provided } (t + a) \neq 0$$

thus $x = \dfrac{3b}{(t+a)^2} + y$ provided $(t + a) \neq 0$

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T21

If $F = \dfrac{Gm_1m_2}{r^2}$ then $r^2 = \dfrac{Gm_1m_2}{F}$

All of the quantities involved are positive, so we can take positive square roots of both sides to find

$$r = \sqrt{\frac{Gm_1m_2}{F}}$$

To check this out with some simple numbers, let $m_1 = 2$, $m_2 = 3$, $r = 4$ and $G = 5$. (This is nothing like the correct value of the constant G, but that does not matter; we are not carrying out a physical calculation, we are just checking algebraic manipulation.) With the assumed values for G, m_1, m_2 and r we find

$$F = \frac{5 \times 2 \times 3}{16} = \frac{30}{16}$$

If these same values are used in the rearranged equation

$$r = \sqrt{\frac{5 \times 2 \times 3}{30/16}} = \sqrt{16} = 4 \qquad \text{which is consistent.}$$

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T22

(a) $V = IR$ where R is a constant

(b) $R = \dfrac{kL}{r^2}$ where k is a constant

(c) $E + \phi = hf$ where h is a constant

(d) $F_{\text{grav}} = \dfrac{Gm_1m_2}{r^2}$ where G is a constant

(Of course you may have used different symbols for the constants.)

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T23

(a), (b), (c), (d) and (e) are true, (f) and (h) are false, (g) is ambiguous since $\sqrt{2} = \pm 1.41$ (This is why some authors prefer to use the root to denote *positive* square roots only.)

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T24

(a) If $2x + 4 > 6$ then $2x > 6 - 4$

i.e. $2x > 2$ so $x > 1$

(b) If $2(a - x) \leq 7$ then $a - x \leq \frac{7}{2}$, i.e. $-x \leq \frac{7}{2} - a$

If both sides are multiplied by -1 and the inequality is reversed $x \geq a - \frac{7}{2}$

(c) If $2(a - x) \leq 3x + b$ then $2a \leq 3x + b + 2x$

i.e. $2a - b \leq 5x$ so $\frac{2a - b}{5} \leq x$

which may be rewritten $x \geq \frac{2a - b}{5}$

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T25

If $R = \dfrac{\rho l}{A}$, then $R > R_0$ implies that $\dfrac{\rho l}{A} > R_0$

(a) It follows that if ρ and A are given $l > \dfrac{R_0 A}{\rho}$

(b) It also follows that if l and ρ are given $A < \dfrac{\rho l}{R_0}$

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